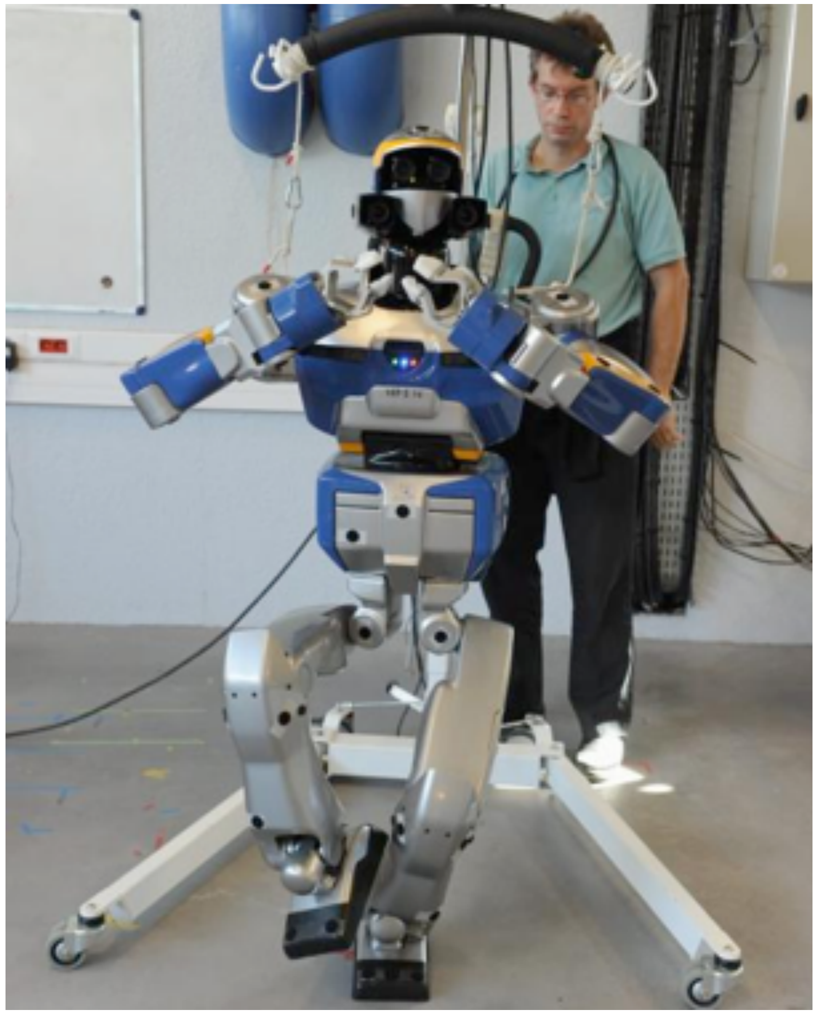
Robust Optimization

for

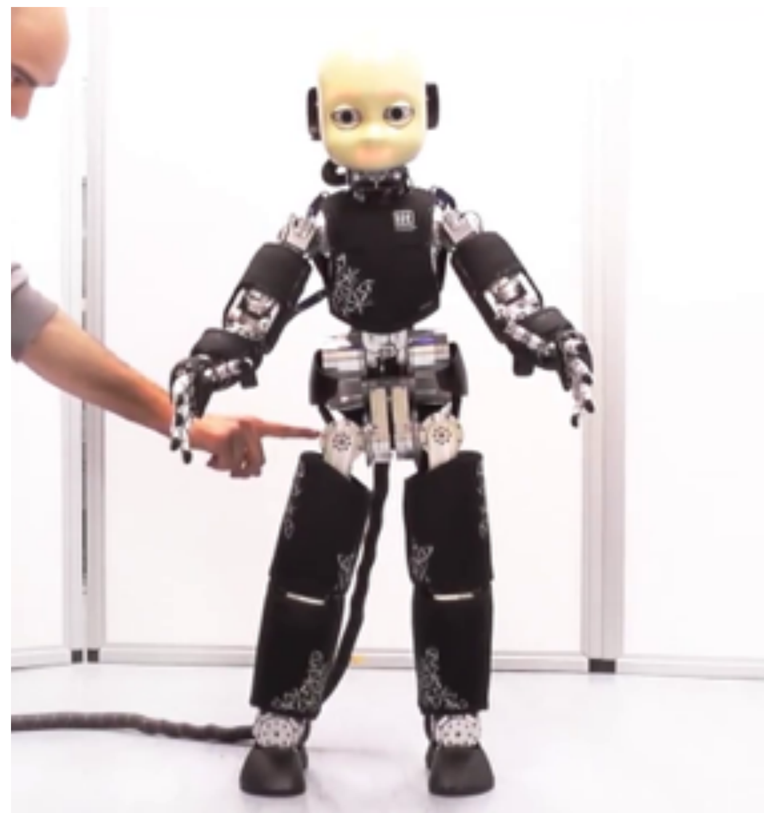
Robust Robotics

Andrea Del Prete, Nirmal Giftsun and Nicolas Mansard

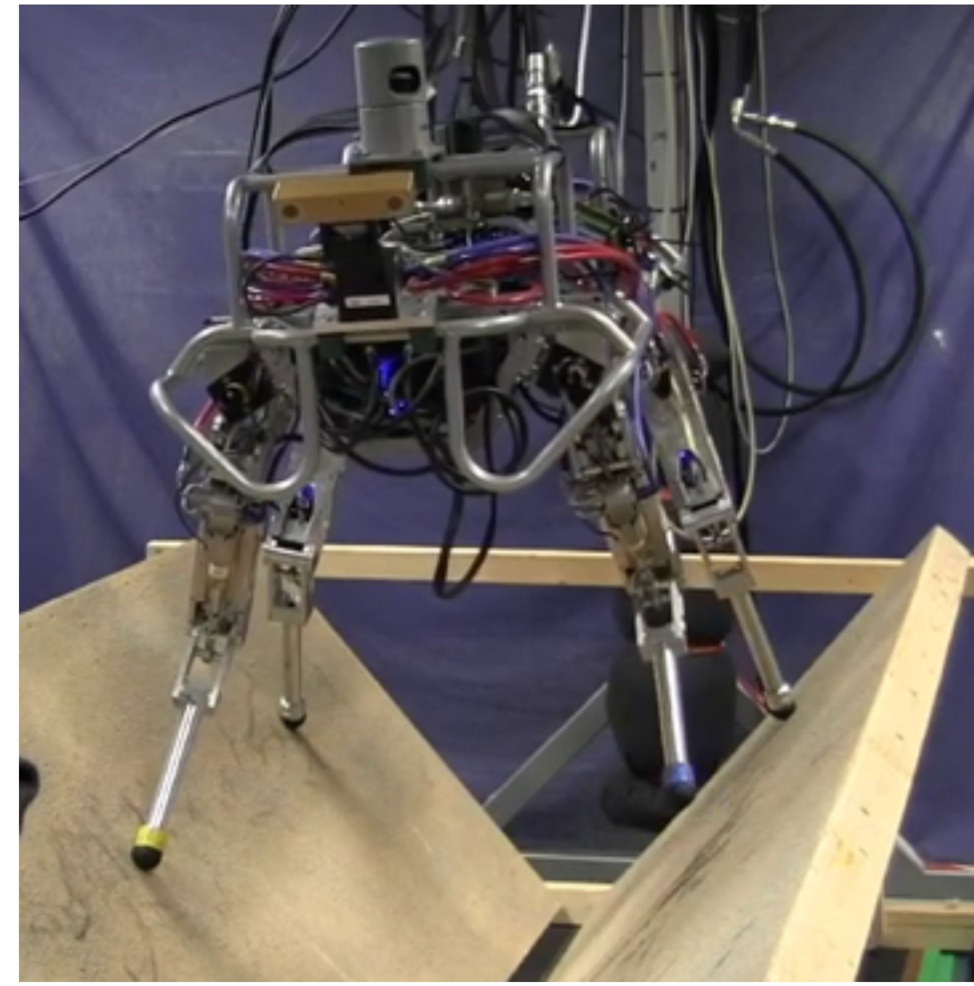
Legged Robots



HRP-2, LAAS



iCub, IIT



HyQ, IIT

Motion Autonomy

Motion Planning & Control Via Numerical optimization

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & c(x, \phi) \\ \text{subject to} \quad & g(x, \phi) \leq 0 \end{aligned}$$

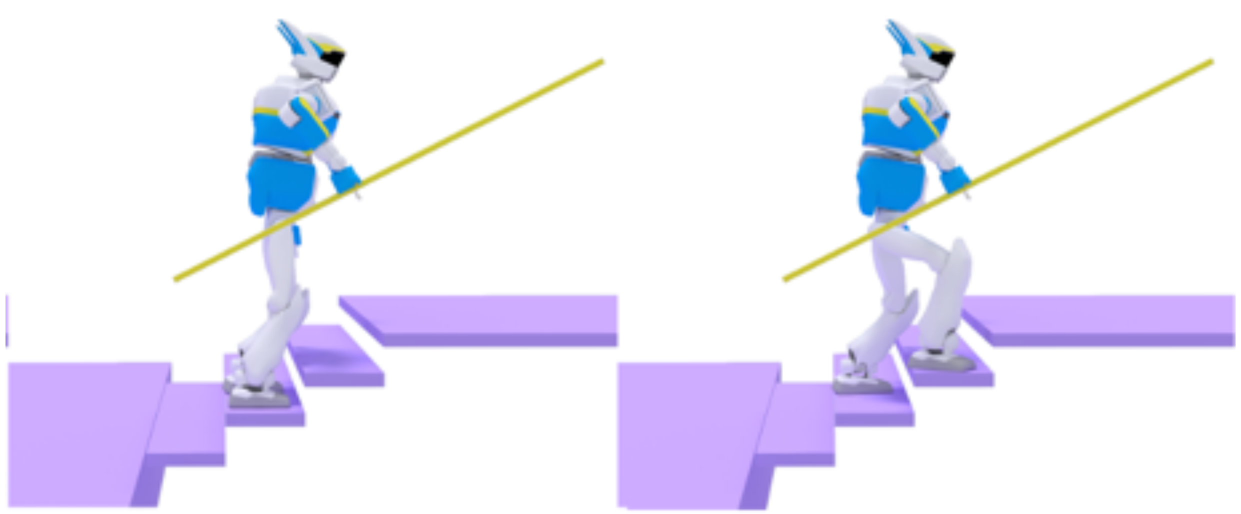
Decision Variables → x

Known Parameters → ϕ

THE PROBLEM

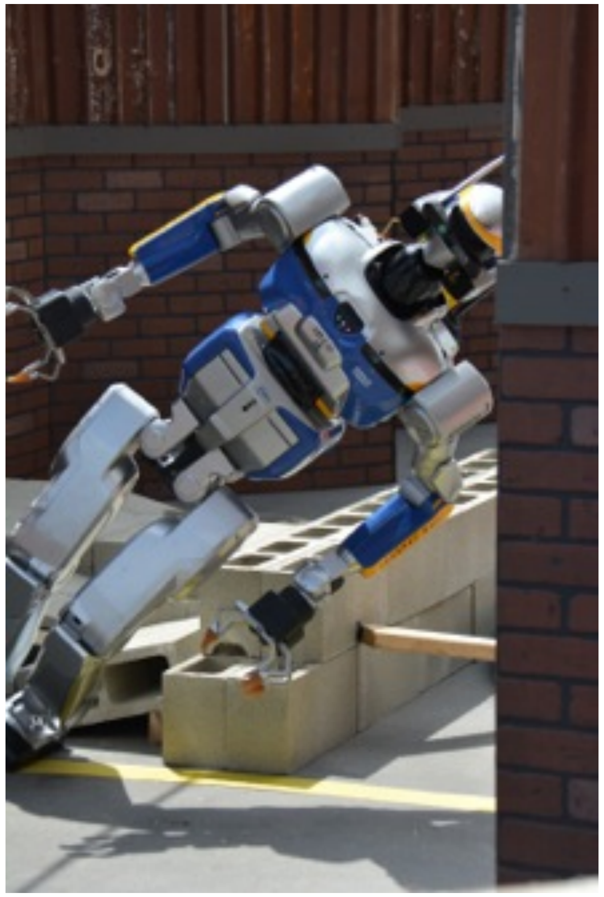
SIMULATION

Tedrake et al. 2014, MIT



Tonneau et al. 2016, LAAS

REALITY



DARPA ROBOTICS CHALLENGE
June 2015, California
16 biped robots participated
15 biped robots fell

State of the Art:

Legged Robots & Uncertainties

Optimization-Based Control	Robust Control
"The faster, the better"	
Tedrake, MIT	
Todorov, Washington Univ.	
Righetti, MPI	

State of the Art:

Legged Robots & Uncertainties

Optimization-Based Control	Robust Control
“The faste the be	“The more robust, the better”
Te MIT	H infinity theory
Tod ashington Univ.	Sliding Mode Control
Righetti, MPI	…

NOT ROBUST

State of the Art:
Legged Robots & Uncertainties

Optimization-Based Control	Robust Control
“The faste the be	“The more ro the be
Te MIT	H inf eory
Tod ashington Univ.	Sl ode Control
Righetti, MPI	…
NOT ROBUST	**NOT APPLICABLE**

Robust Model Predictive Control

Take-Home Message

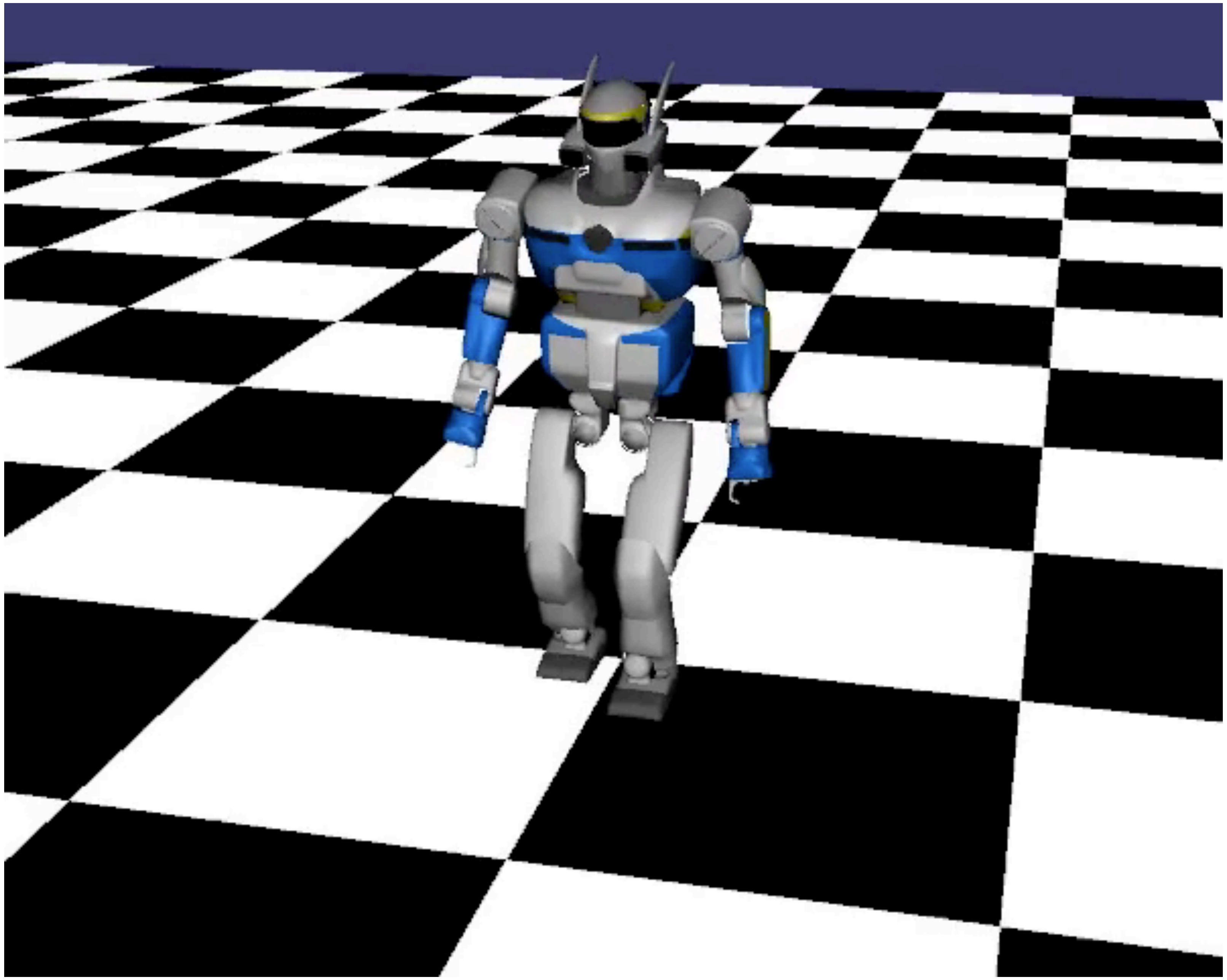
$$\begin{aligned} \underset{x}{\text{minimize}} \quad & c(x, \phi) \\ \text{subject to} \quad & g(x, \phi) \leq 0 \end{aligned}$$

Decision Variables

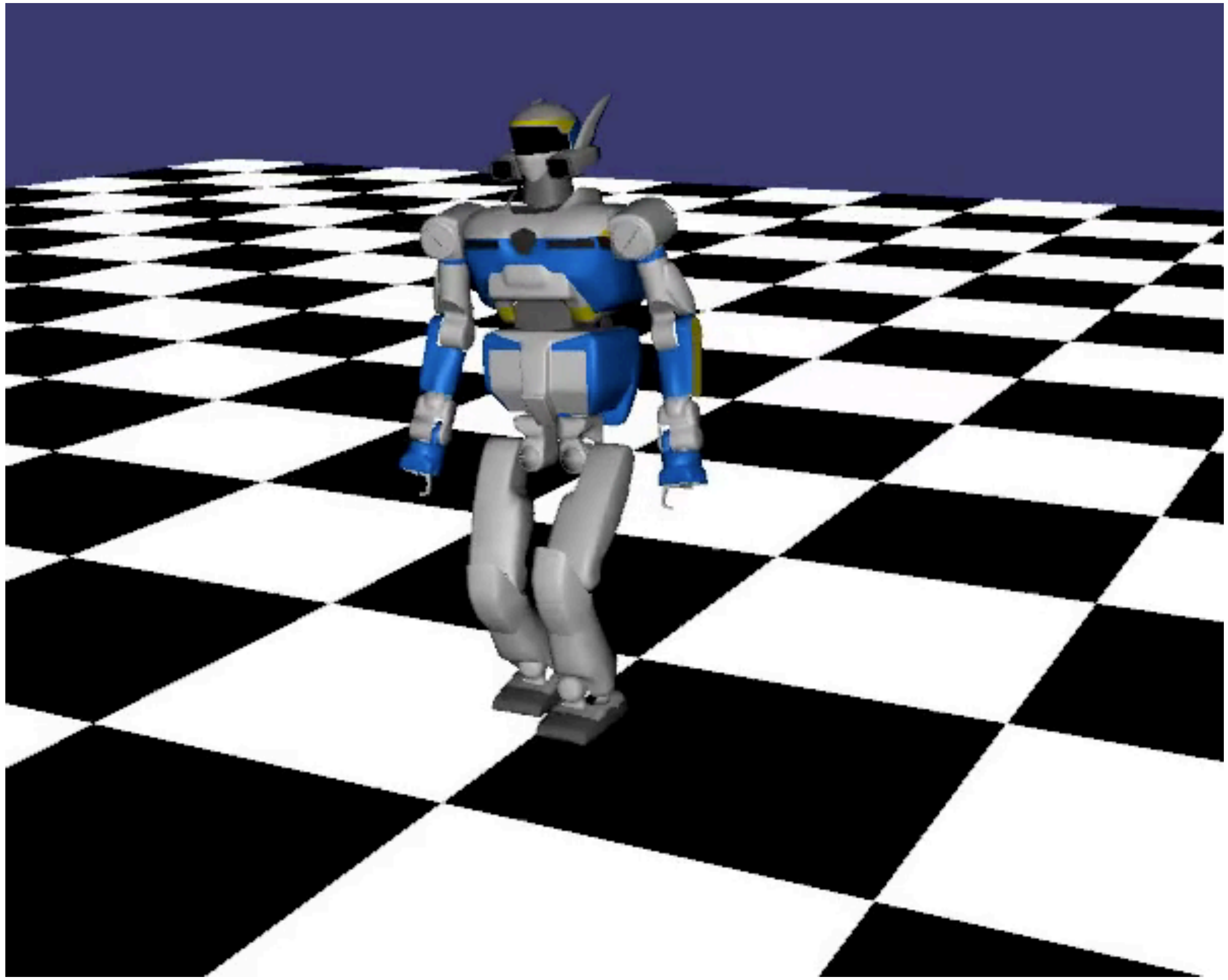
Uncertain Parameters

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & \left[\max_{\phi \in \mathcal{H}} \quad c(x, \phi) \right] \\ \text{subject to} \quad & g(x, \phi) \leq 0 \quad \forall \phi \in \mathcal{H} \end{aligned}$$

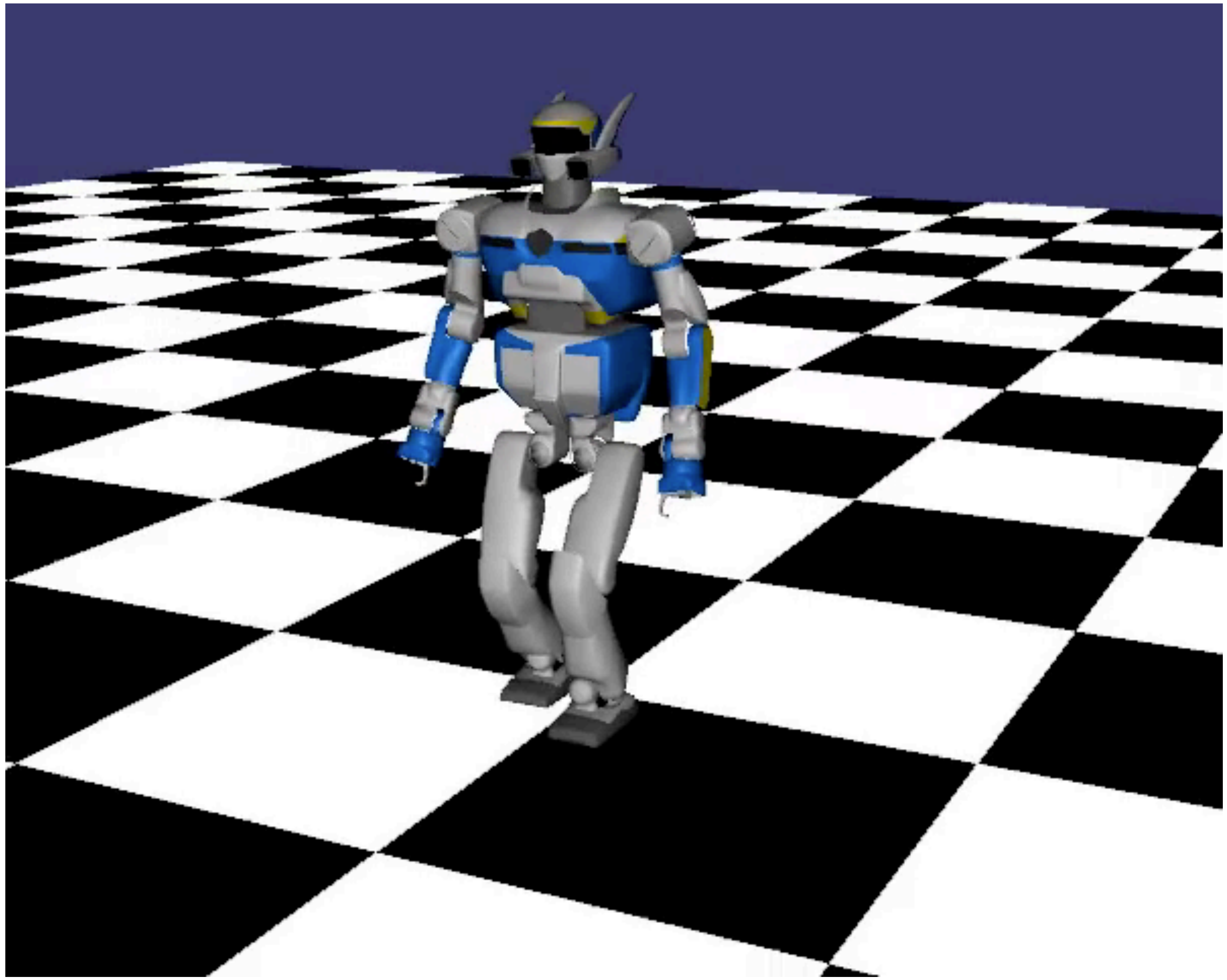
WALKING IS EASY!



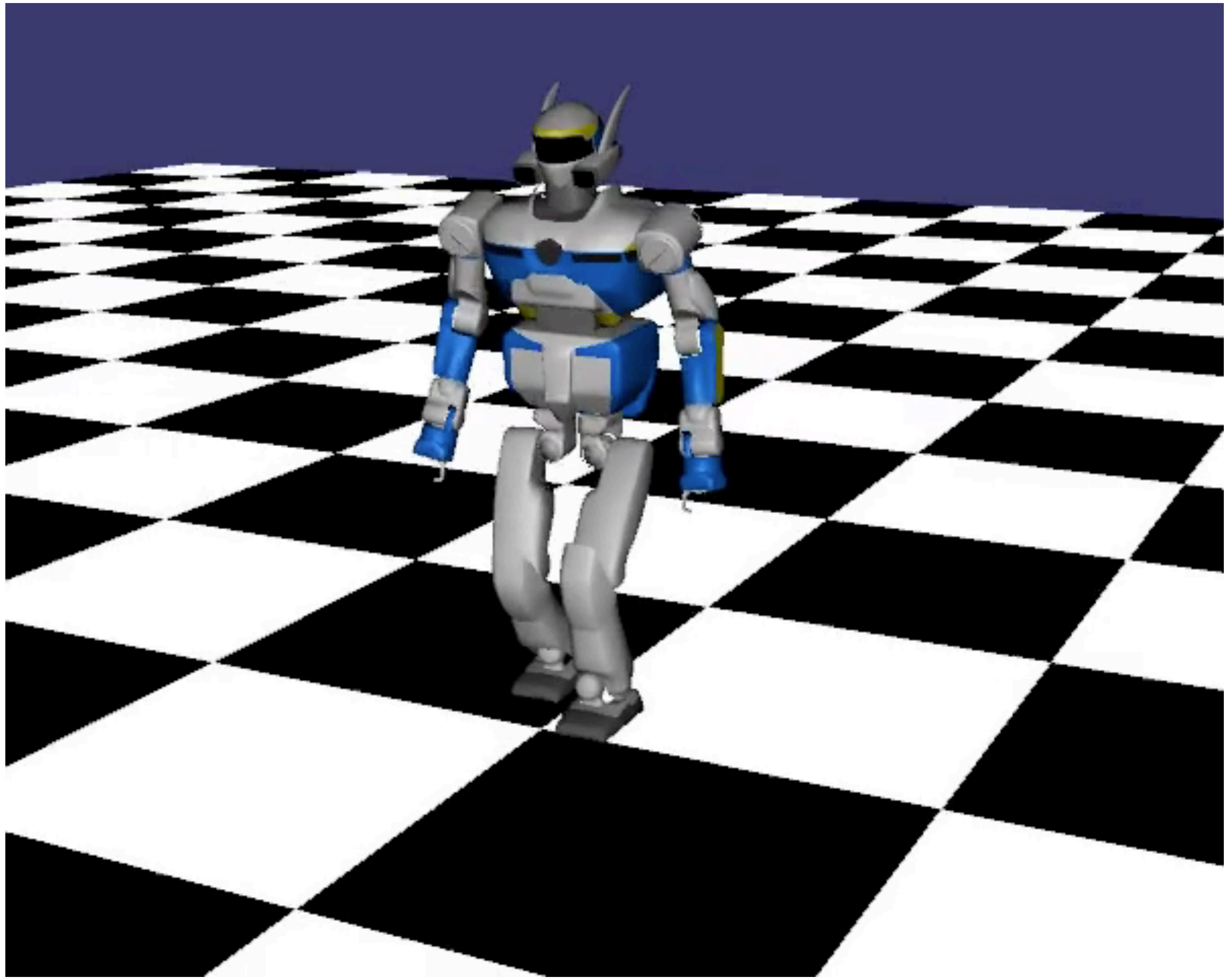
…in a controlled simulation environment with no noises, no modeling errors and no delays



But what if we add noise to the joint torques?



…or if we add delays in the velocity estimation?



…or if we introduce errors in the inertial parameters of the robot?

We propose to use robust optimization to design controllers that are robust to uncertainties in the joint torques

Least-Squares Optimization

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & ||Ax - a||^2 \\ \text{subject to} \quad & Bx + b \geq 0 \end{aligned}$$

LEAST-SQUARES OPTIMIZATION

$$\underset{x}{\text{minimize}} \quad ||Ax - a||^2$$

$$\text{subject to} \quad \cancel{Bx + b \geq 0}$$

$$B(x + e) + b \geq 0$$

Torque tracking error

Least-Squares Optimization

$$\underset{x}{\text{minimize}} \quad ||Ax - a||^2$$

$$\text{subject to} \quad Bx + b \geq 0$$

$$B(x + e) + b \geq 0$$

Torque tracking error

Option 1

$$e \sim \mathcal{N}(0, \Sigma)$$

$$P(B(x + e) + b \geq 0) \geq 0.99$$

Least-Squares Optimization

$$\underset{x}{\text{minimize}} \quad ||Ax - a||^2$$

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Torque tracking error

Option 1

$$e \sim \mathcal{N}(0, \Sigma)$$

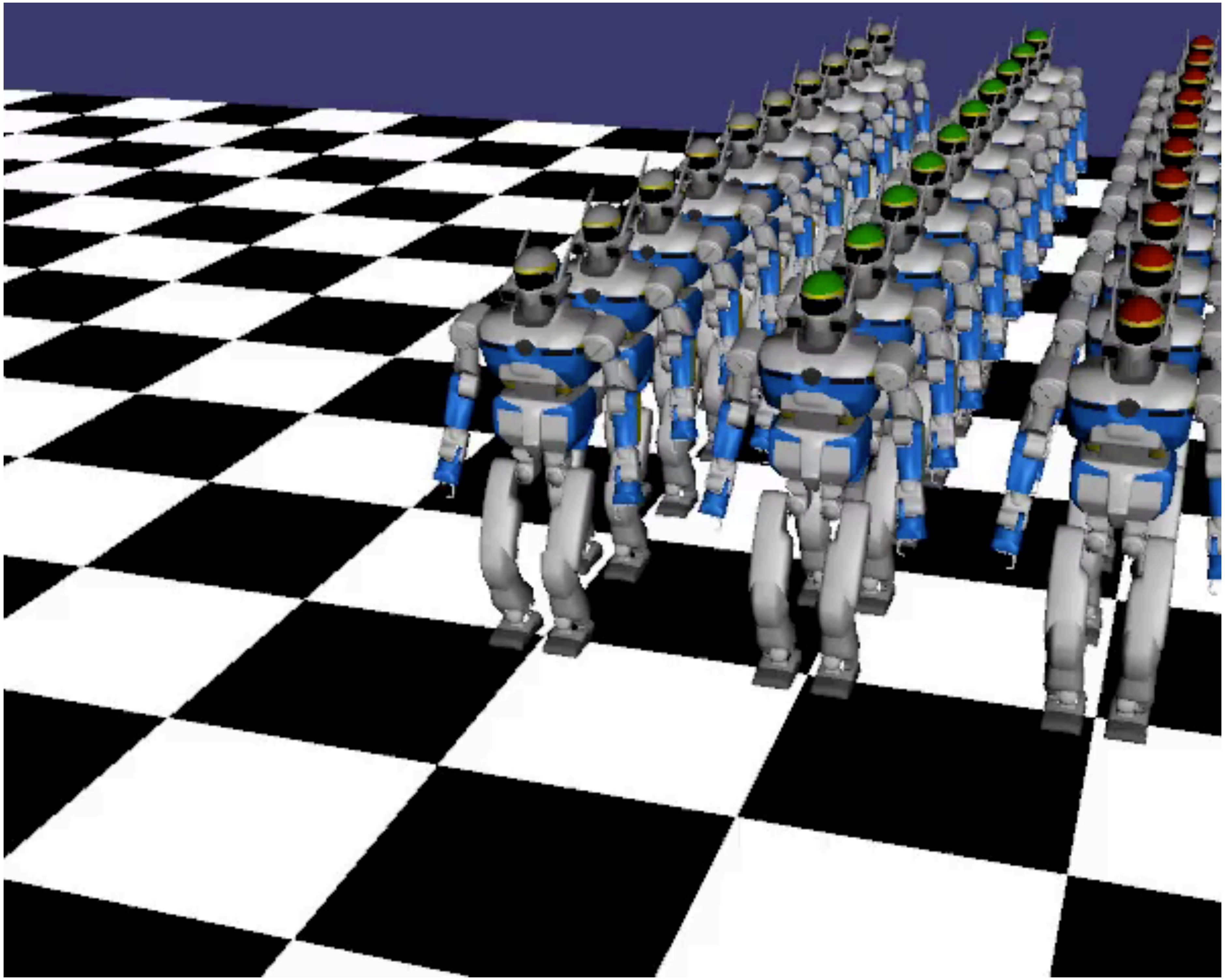
$$P(B(x + e) + b \geq 0) \geq 0.99$$

Option 2

$$|e| \leq e^{max}$$

$$B(x + e) + b \geq 0 \qquad \forall e$$

0



Classic

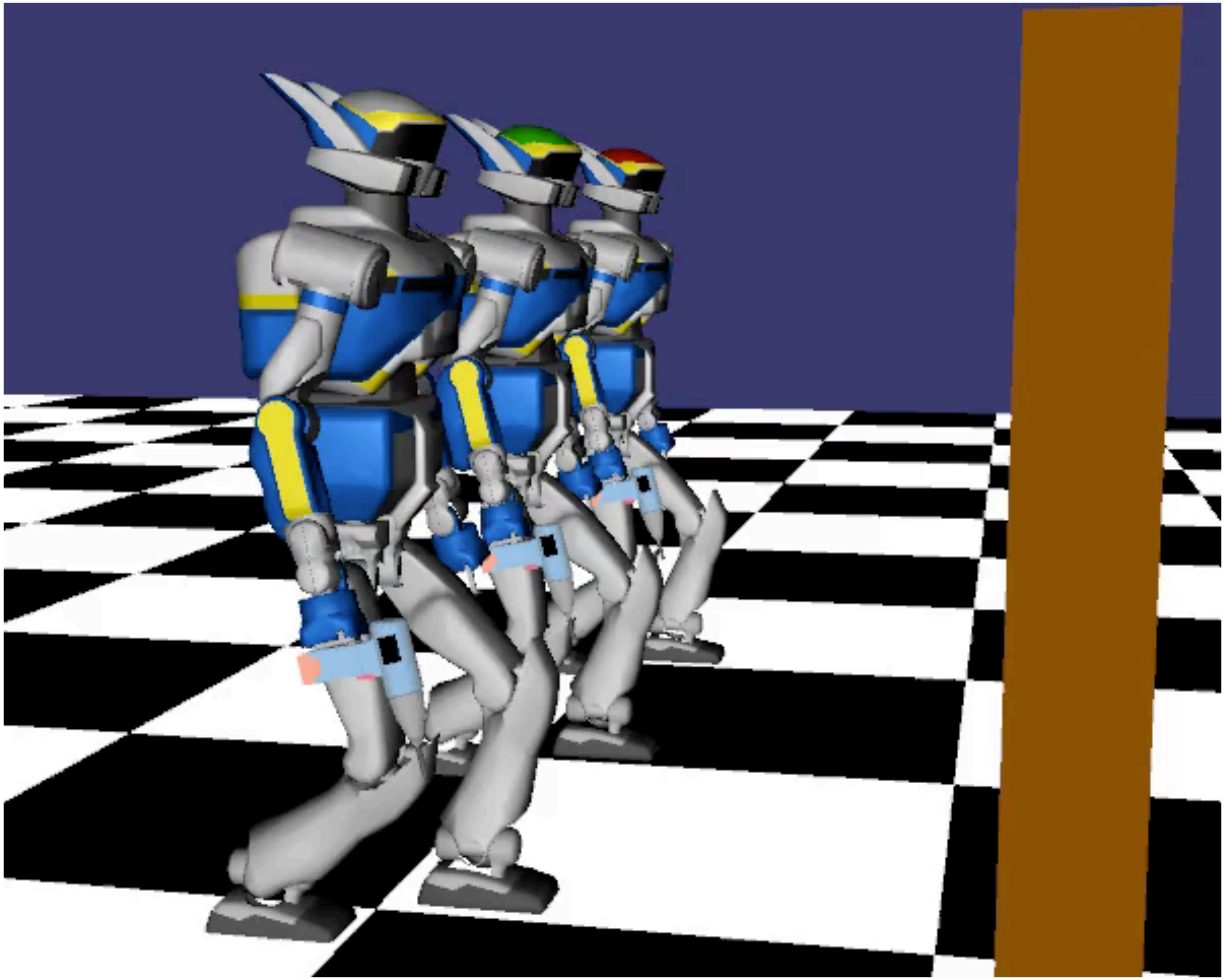
Robust Stochastic

Robust Worst-Case

RESULTS

Uncertainties						Mean time before falling [s]			Number of falls		
v	$\frac{\sigma}{\tau^{max}}$[%]	Torque bandwidth [Hz]	mass [%]	com [cm]	inertia [%]	Classic	Robust p_{ind}	Robust p_{box}	Classic	Robust p_{ind}	Robust p_{box}
Real	0	∞	0	0	0	∞	∞	∞	0	0	0
Estimated	0	∞	0	0	0	∞	∞	∞	0	0	0
Real	0	20	0	0	0	∞	∞	∞	0	0	0
Estimated	0	20	0	0	0	16.8	∞	20.5	100	0	100
Real	6	∞	0	0	0	203.2	∞	∞	20	0	0
Estimated	6	∞	0	0	0	109.4	912.1	202.5	39	4	20
Estimated	8	∞	0	0	0	69.0	408.3	105.3	58	10	38
Real	6	20	0	0	0	172.6	908.4	∞	23	5	0
Estimated	6	20	0	0	0	24.7	147.3	35.8	98	28	80
Real	0	∞	10	1	20	282.1	∞	∞	15	0	0
Estimated	6	∞	0	0	20	106.1	921.2	240.4	40	4	17
Estimated	6	∞	0	0	100	109.1	761.4	187.2	39	5	22
Estimated	6	∞	10	1	20	94.0	765.7	100.3	44	5	38
Estimated	8	∞	10	1	20	59.0	316.1	102.1	65	14	40
Estimated	5	20	10	1	20	30.8	148.2	33.7	90	28	79

Drilling Task



Classic

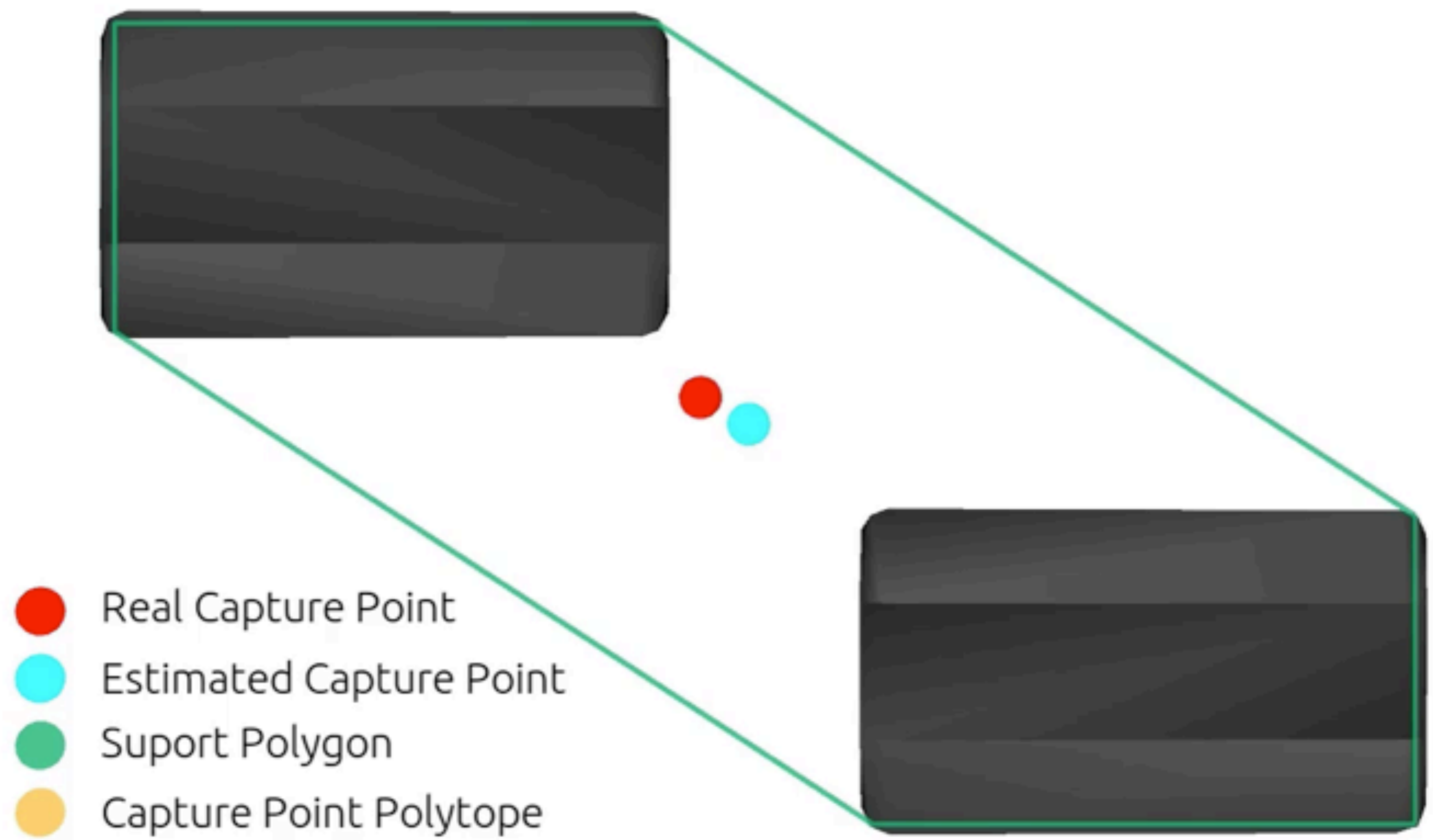
Robust Stochastic

Robust Worst-Case

And what about uncertainties in the **inertial parameters**?

Joint work with Nirmal Giftsun, PhD candidate

Inertial Parameter Robustness



Inertial Parameter Robustness

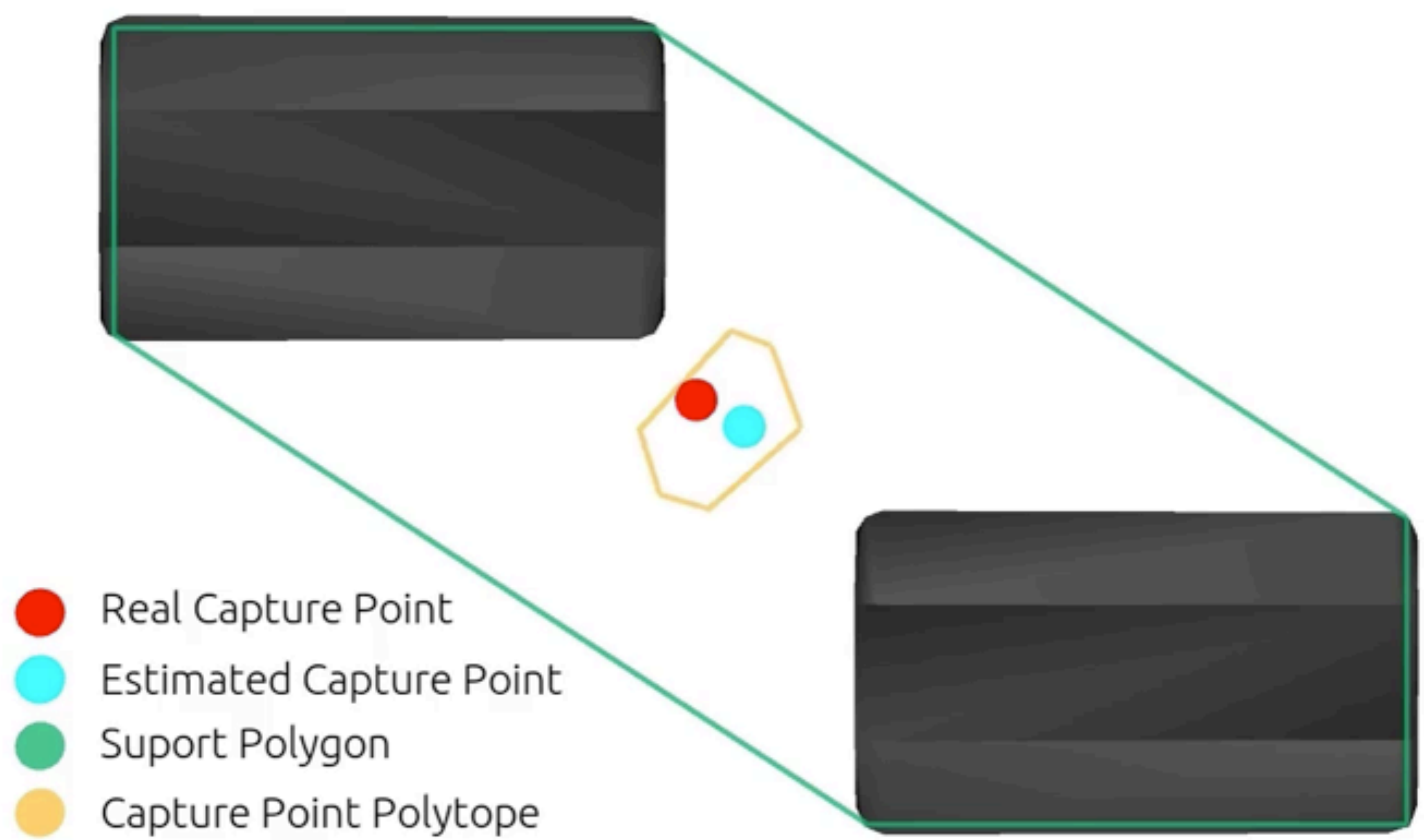
$$\phi_i = (m_i, m_i{}^i c_i, I_i^{xx}, I_i^{xy}, I_i^{xz}, I_i^{yy}, I_i^{yz}, I_i^{zz})$$

$$\phi = (\phi_1, \ldots, \phi_N)$$

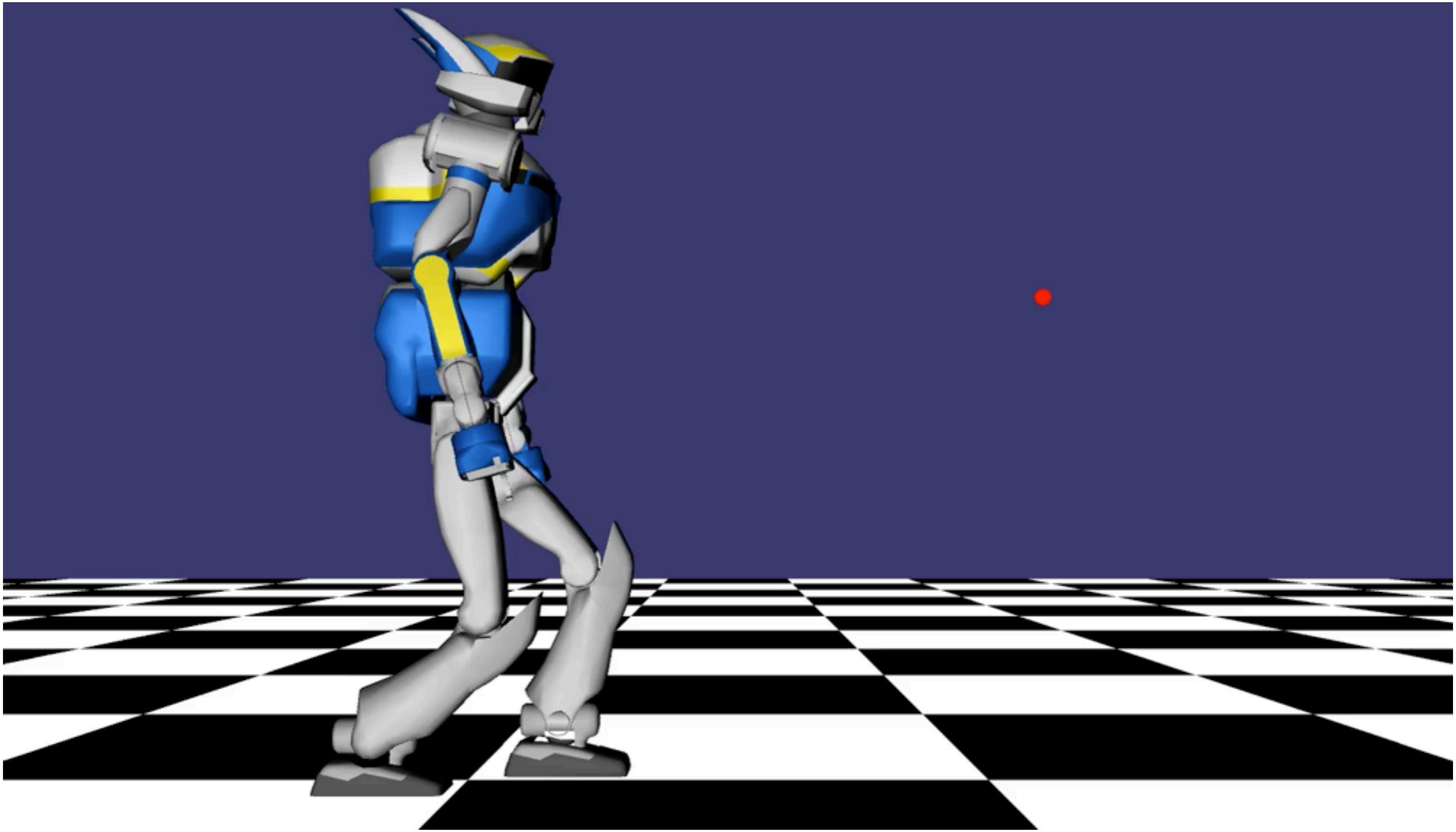
$$\begin{aligned} c^{xy} &= P\frac{\sum_{i=1}^{N} m_i(p_i + {}^w R_i{}^i c_i)}{m_{tot}} \\ &= \sum_{i=1}^{N} \underbrace{m_{tot}^{-1} P \begin{bmatrix} p_i & {}^w R_i & 0_{3\times 6} \end{bmatrix}}_{F_i} \phi_i \\ &= \begin{bmatrix} F_1 & \ldots & F_N \end{bmatrix} \phi = F\phi, \end{aligned}$$

where $P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

Inertial Parameter Robustness

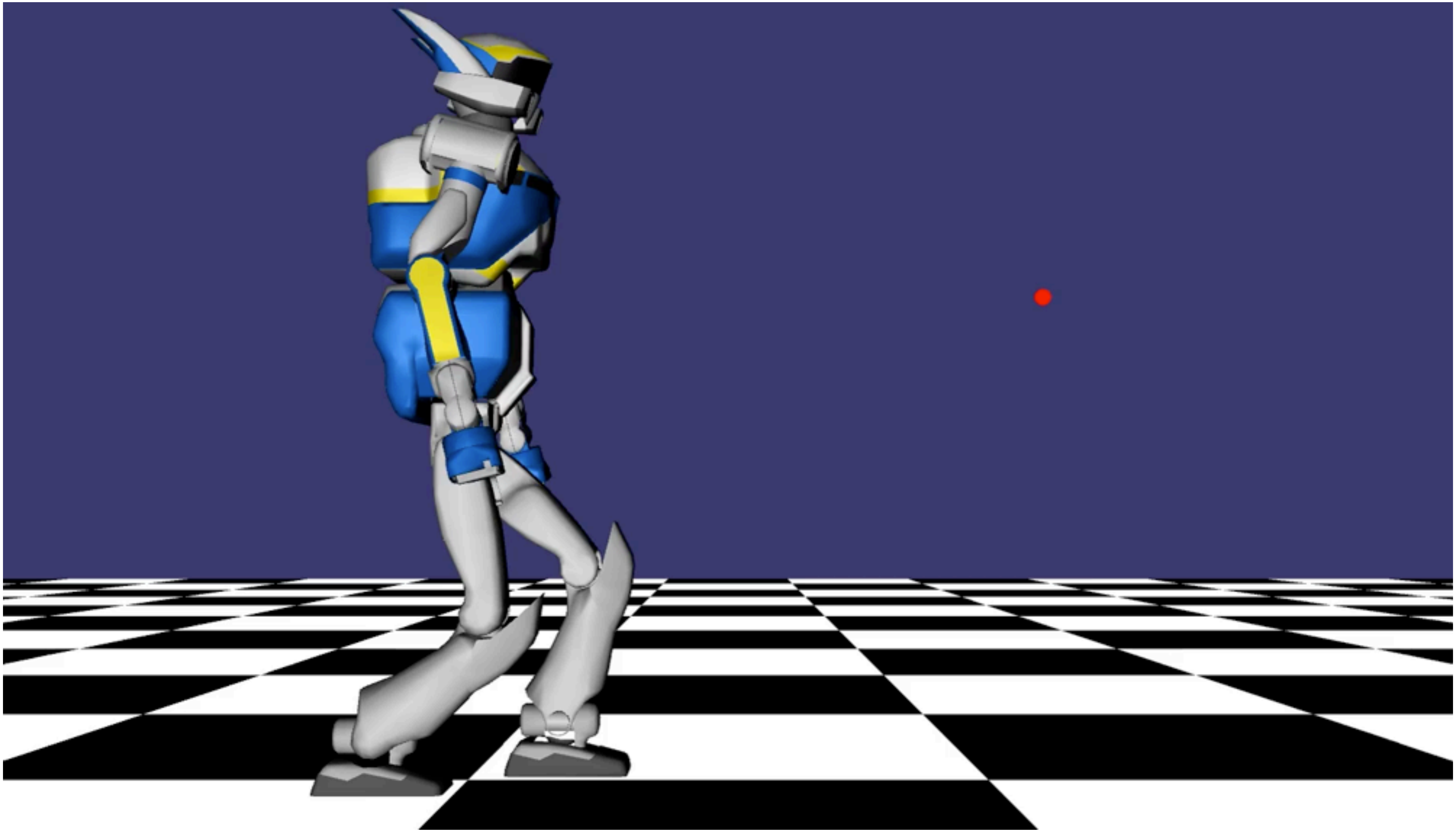


Inertial Parameter Robustness



Classic Controller

Inertial Parameter Robustness



Robust Controller

Inertial Parameter Robustness

Unreachable Target

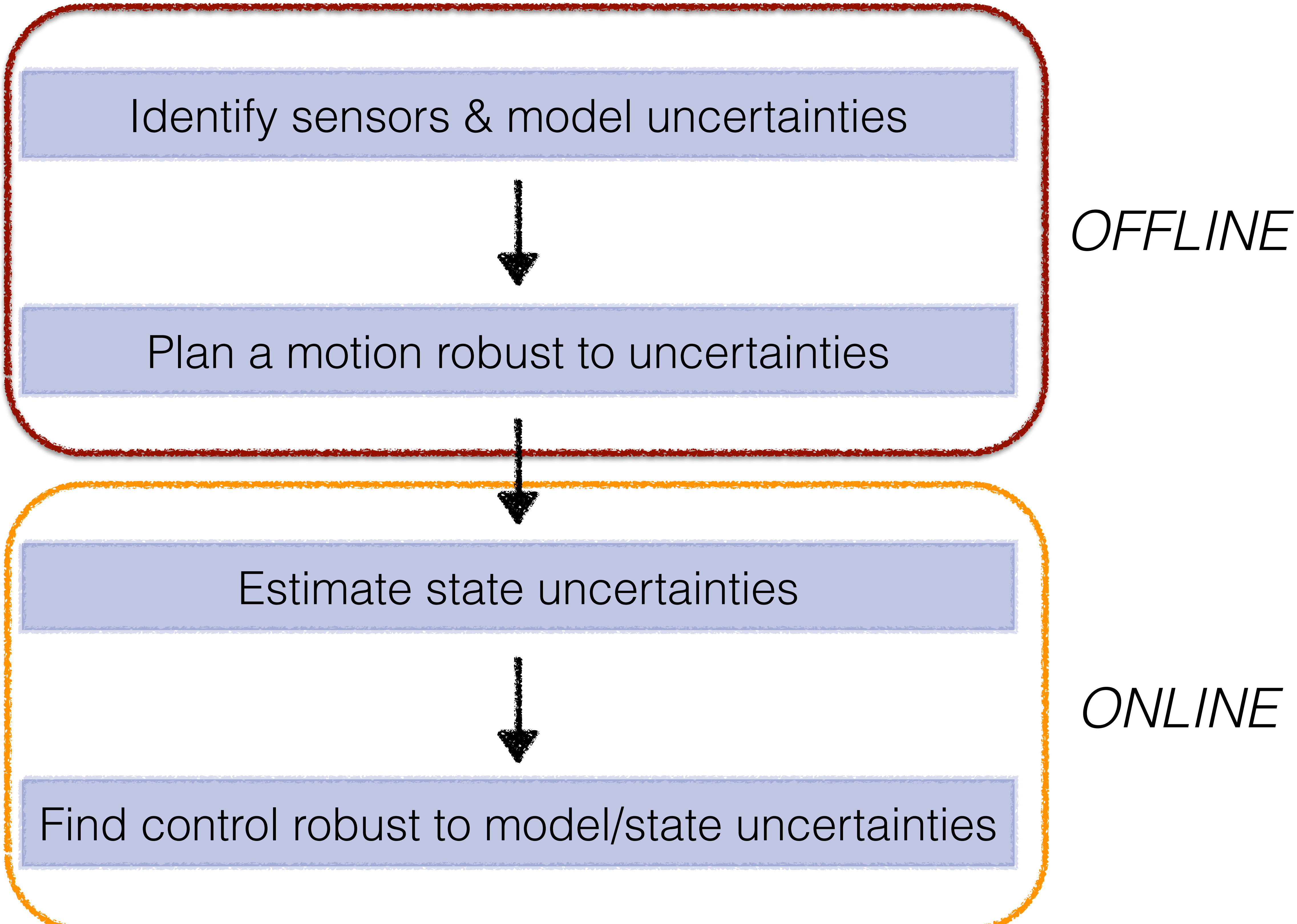
Uncertainties		Classic Controller			Robust Controller		
Max Mass Error [%]	Max CoM Error[mm]	Falls	Task Time [s]	Task Error [mm]	Falls	Task Time [s]	Task Error [mm]
10	12.5	35	6.22	4	1	5.26	5
10	25	31	6.30	5	0	7.32	12
10	50	43	4.49	70	0	4.66	110
30	50	43	4.48	70	0	4.68	110
30	100	40	5.44	30	4	5.81	80

Reachable Target

Uncertainties		Classic Controller			Robust Controller		
Max Mass Error [%]	Max CoM Error[mm]	Falls	Task Time [s]	Task Error [mm]	Falls	Task Time [s]	Task Error [mm]
10	12.5	59	3.64	0	5	3.06	0
10	25.0	42	3.40	0.18	4	2.94	0.12

And what about the other uncertainties?

Robust Robotics

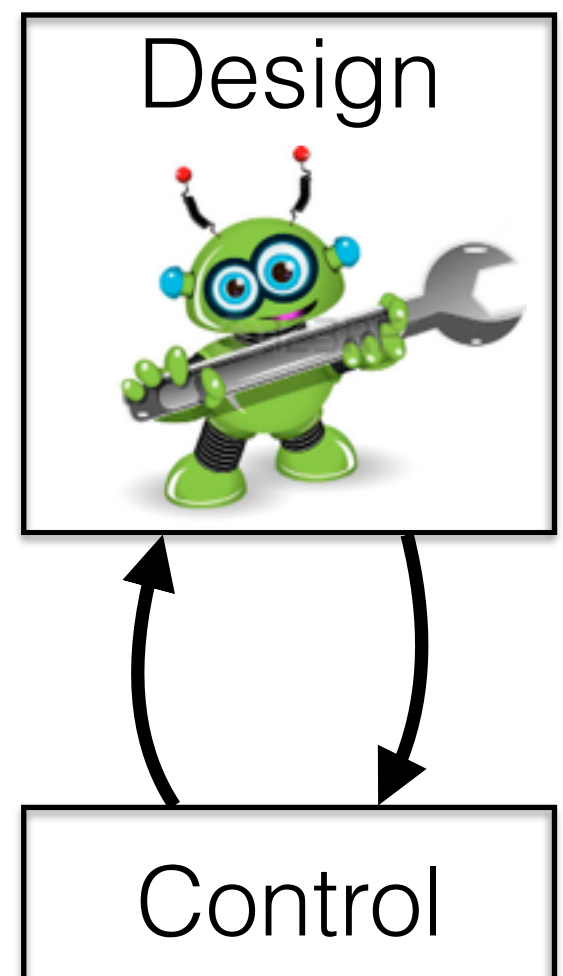


Expected Results

Improve performance

Provide performance guarantees

Identify hardware bottlenecks



Take-Home Message

~~$$\begin{aligned} \underset{x}{\text{minimize}} \quad & c(x, \phi) \\ \text{subject to} \quad & g(x, \phi) \leq 0 \end{aligned}$$~~

The End

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & \left[\max_{\phi \in \mathcal{H}} \quad c(x, \phi) \right] \\ \text{subject to} \quad & g(x, \phi) \leq 0 \quad \forall \phi \in \mathcal{H} \end{aligned}$$

The End

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